

## TIME-EVOLUTION OF SPHERICAL DROPLETS IN VAPOUR ENVIRONMENT: A PHYSICAL–MATHEMATICAL MODEL

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**Abstract**—In the present paper a physical–mathematical model for the process of heat and mass transfer at the interface of a liquid–vapour system is proposed. On the basis of this model and by means of the conservation equations of mass and energy fluxes, the evolution-equations of the state of a single droplet in continuum environment are determined. These equations are ordinary non-linear differential equations which have been numerically integrated in order to give comparisons with experimental results of other authors.

### NOMENCLATURE

$c_s$ , specific heat;  
 $d$ , number of degree of freedom;  
 $k$ , Boltzmann constant;  
 $L$ , latent heat of vaporization;  
 $m$ , mass of a molecule;  
 $n$ , numerical density;  
 $N$ , number flux;  
 $R$ , radius of a droplet;  
 $s$ , speed ratio;  
 $T$ , temperature;  
 $V$ , incident velocity of a mol;  
 $W$ , drift velocity.

$\mu$ , =  $\gamma \cdot \tau$ ;  
 $\nu$ , =  $n_e/n_i$ ;  
 $\tau$ , =  $T_a/T$ ;  
 $\omega$ , highest velocity in the distribution function of the condensing molecules.

### Subscripts

$a$ , ambient;  
 $c$ , critical temperature;  
 $e$ , equilibrium;  
 $i$ , incident;  
 $l$ , liquid-phase;  
 $r$ , re-emission;  
 $s$ , solidification temperature.

### Greek symbols

$\gamma$ , =  $n_a/n_e$ ;  
 $\delta$ ,  $\begin{cases} = 0 & \text{monoatomic gas;} \\ = 1 & \text{biatomic gas;} \\ = 3/2 & \text{polyatomic gas;} \end{cases}$

### Superscripts

$+$ , vaporization;  
 $-$ , condensation.

### 1. INTRODUCTION

THE VAPORIZATION of a spherical droplet into semi-infinite vapour space is a well known diffusion problem with free-boundary (the interface between liquid and vapour) and can be studied as a classical Stephan problem, see ref. [1] and related bibliography. Moreover when the dimensions of the droplet itself can be compared with the mean-free-path in the vapour a further difficulty arises; in fact the physical–mathematical model of the continuum mechanics should be substituted, in the vapour phase, by the Boltzmann equation with suitable matching, on the moving boundary, of this equation with the parabolic ones governing the heat transfer in the liquid phase.

In spite of the fact that this problem has been studied by many authors, either with the equations of continuum mechanics (see, amongst the others, Nix and Fukuta [2], Vdovin *et al.* [5], Chien [6] and Torda [7]) or with the equations of kinetic theory in rarefied gas conditions (see Matsushita [3] and Bellomo [9]), at present the proposed models have a limited range of validity in the sense that they can supply reliable results only for predominant vaporization in rarefied flow or in conditions of small perturbation from the equilibrium ones (see ref. [8]).

In this paper we shall propose a physical–mathematical model describing the heat and mass transfer of a two-phase system. The model is based on a recently proposed phase-transition kinetic theory [9], which has given theoretical results very close to the experimental ones, and on the mathematical methods of kinetic theory [10]. The model gives the time-evolution of the state of a droplet in terms of an ordinary non-linear differential equation: this equation is here discussed by an analytical and numerical point of view.

The obtained results are compared with the experimental ones of other authors [11].

## 2. PHYSICAL-MATHEMATICAL MODEL OF HEAT AND MASS TRANSFER

Let us take into account a spherical droplet with radius  $R$  and mean inner temperature  $T$ , suspended in a vapour environment of temperature  $T_a$  and number density  $n_a$ . The aim of this work is finding: (1) a physical-mathematical model describing the heat and mass transfer and (2) an evolution equation, in terms of an ordinary differential equation, on the state variables  $T$  and  $R$ .

This mathematical modeling can be achieved with the methods of kinetic theory of gases according to the following hypothesis:

- (a) The vapour molecules impinge the surface of the droplet with a distribution function of maxwellian type with drift velocity:

$$f_i = n_a \cdot \exp\{-|\mathbf{V} - \mathbf{W}|^2 / (2k/m \cdot T_a)\}. \quad (1)$$

In particular, in equilibrium conditions:

$$W = 0, \quad T_a = T, \quad n_a = n_e \Rightarrow f_e = n_e \cdot \exp\{-V^2 / (2k/m \cdot T)\}. \quad (2)$$

- (b) The number of molecules which condense or vaporize in the unit surface and in the unit time, respectively  $N^-$  and  $N^+$ , are given as follows, according to [9]:

$$N^- = N_i \cdot \phi; \quad N^+ = N_e \cdot \phi, \quad \phi \in [0, 1]. \quad (3)$$

- (c)  $\phi = \phi(T)$  is the condensation/vaporization function, as it follows from the kinetic theory of phase-transition and, according to ref. [9], is assumed to be a function, for a given vapour-liquid pair, only of the temperature of the liquid phase.

- (d) The gradients of temperature or density inside the liquid phase are negligible.

- (e) The re-emitted molecules (vaporized or scattered) are in equilibrium at the temperature  $T$ .

With regard to these hypotheses let us remark that equation (1) gives a model-solution of the Boltzmann equation and, once  $W$ , at fixed values of  $n_a$  and  $T_a$ , has been found, can give an adequate approximation. From equation (1) the unit number and energy fluxes, according to known results of the kinetic theory [10, 12], can be calculated as follows:

$$N_i = \int_{\mathbf{V} \cdot \mathbf{n} \leq 0} |\mathbf{V} \cdot \mathbf{n}| \cdot f_i \, dV = \frac{n_a}{2(\pi)^{1/2}} (2k/m \cdot T_a)^{1/2} \cdot \chi(s), \quad (4)$$

$$E_i = \int_{\mathbf{V} \cdot \mathbf{n} \leq 0} \frac{1}{2} m V^2 |\mathbf{V} \cdot \mathbf{n}| \cdot f_i \, dV = \frac{n_a}{2(\pi)^{1/2}} (2k/m \cdot T_a)^{1/2} \cdot k T_a \cdot f(s), \quad (5)$$

$$s = W / (2k/m \cdot T_a)^{1/2}; \quad \chi = \exp(-s^2) + (\pi)^{1/2} \cdot s [1 + \operatorname{erf}(s)], \quad (6)$$

$$f = (s^2 + 2 + \delta) \cdot \chi + \frac{(\pi)^{1/2}}{2} s [1 + \operatorname{erf}(s)]. \quad (7)$$

In particular, in equilibrium conditions [ $s = 0$ ,  $n_e = n_e(T)$ ], equations (4 and 5) become:

$$N_e = \frac{n_e}{2(\pi)^{1/2}} (2k/m \cdot T)^{1/2} \quad (8)$$

$$E_e = \frac{n_e}{2(\pi)^{1/2}} (2k/m \cdot T)^{1/2} \cdot k T \cdot (2 + \delta). \quad (9)$$

The number fluxes of vaporized and condensed molecules are given by equation (3), where (see [9]):

$$\phi(T) = \{\exp(-y) - 1 - y\} / \exp(y); \quad y = y(T) = \omega(T) / (2k/m \cdot T)^{1/2}, \quad (10)$$

$y$  has the physical meaning given in [9], namely all vapour molecules hitting the surface of the liquid accommodate in maxwellian equilibrium at the temperature of the surface and condense if  $V \leq \omega$ , whereas they are re-emitted if  $V > \omega$ ; according to [9],  $y$  can be calculated from:

$$\left[ \frac{3}{2} - \frac{6}{d} L(T) \right] / (2k/m \cdot T) = \int_0^y \eta^4 \exp(-\eta^2) \, d\eta / \int_0^y \eta^2 \exp(-\eta^2) \, d\eta. \quad (11)$$

Moreover, according to hypotheses (d) and (e), the re-emitted energy flux is given by:

$$E_r = N_i \cdot k T (2 + \delta). \quad (12)$$

According to the analytical manipulation of equations (4, 5, 8 and 9) let us now define the following dimensionless functions:

$$\psi_N \stackrel{\text{df}}{=} N_i/N_e = \gamma \cdot \tau^{1/2} \cdot \chi; \quad \gamma = n_d/n_e; \quad \tau = T_d/T \quad (13)$$

$$\psi_E \stackrel{\text{df}}{=} E_i/E_e = \gamma \cdot \tau^{3/2} \cdot f/(2 + \delta). \quad (14)$$

The equations (4–14) give the heat- and mass-transfer physical model once a mathematical expression for the speed ratio  $s$  has been found according to the following limiting physical conditions of the system:

- (1)  $\gamma = \tau = 1 \Rightarrow \psi_N = \psi_E = 1 \Rightarrow s = 0$  (equilibrium conditions);
- (2)  $\forall \tau$  and  $\gamma \rightarrow \infty \Rightarrow \psi_N \rightarrow N_d/N_e = \gamma \cdot \tau^{1/2}$ ,  $\psi_E \rightarrow E_d/E_e = \gamma \cdot \tau^{3/2} \Rightarrow s \rightarrow 0$   
(condensation with negligible vaporization);
- (3)  $\forall \tau$  and  $\gamma \rightarrow 0 \Rightarrow \psi_N = \psi_E \rightarrow 0 \Rightarrow s \rightarrow s_{\min} < 0$  (vaporization into vacuum)  
( $s$  is taken negative if the drift velocity is directed from the liquid into the vapour).

From these limiting conditions it follows that a conceivable model for the speed ratio is to assume  $s = s(\mu)$ ,  $\mu = \gamma \cdot \tau$ , according to the following further hypotheses:

- (f) when vaporization into vacuum occurs, the drift velocity equals the normal mean velocity; from [12] follows:

$$\frac{\langle W \rangle}{(2k/m \cdot T_a)^{1/2}} = \frac{1}{\pi^{3/2}(2k/m \cdot T_a)^2} \int_{\mathbf{V} \cdot \mathbf{n} \leq 0} |\mathbf{V} \cdot \mathbf{n}| \exp\{-V^2/(2k/m \cdot T_a)\} d\mathbf{V} = \frac{1}{2(\pi)^{1/2}}$$

and consequently:  $s_{\min} = -1/[2(\pi)^{1/2}]$ .

- (g)  $ds/d\mu|_{\mu=0} = 0$ .

- (h)  $\forall \mu > 1 \Rightarrow s \geq 0$  and  $\exists \mu^* > 0: s(\mu^*) = s_{\max} = 1/[2(\pi)^{1/2}]$ .

One of the possible functions which is consistent with the previous constraints is the following:

$$s = \frac{1}{2(\pi)^{1/2}} (\mu^2 - 1) \cdot \exp\{-\mu^2/3.62\}. \quad (15)$$

The behaviour of  $s$  vs  $\mu$  is visualized in Fig. 1. The set of equations (4–15) gives the announced heat and mass transfer model.

Finally let us calculate the mass and energy fluxes, respectively  $\Phi_N$  and  $\Phi_E$ , from the vapour volume into the spherical droplet; according to equations (3, 4, 5, 10 and 12), taking also into account the dimensionless quantities of equations (13 and 14), the following expressions can be obtained:

$$\Phi_N = 4\pi R^2 m (N^- - N^+) = 4\pi R^2 N_e \cdot \phi(\psi_N - 1), \quad (16)$$

$$\Phi_E = 4\pi R^2 \{(E_i - E_e) + (E^- - E^+)\} = 4\pi R^2 N_e kT (2 + \delta) \cdot [\phi(\psi_N - 1) + (\psi_E - \psi_N)]. \quad (17)$$

Equation (17) takes into account the balance between the energy flux due to collisions and the one due to mass transfer (condensation–vaporization), with the assumption, according to hypothesis (e), of complete accommodation of vapour molecules before re-emission or vaporization/condensation at the temperature  $T$  of the droplet.

### 3. EQUATIONS OF EVOLUTION OF A SPHERICAL DROPLET

Let us consider now a spherical droplet in non-equilibrium conditions in a vapour environment. The state of a single droplet is then defined by its radius  $R = R(t)$  and by its mean temperature  $T = T(t)$ , while the state of the outer system is defined by the temperature  $T_a$  and the number density  $n_a$  of the vapour in the ambient far from the droplet.

Let us comment that a time variation of  $T_a$  and  $n_a$  can be considered as due to changes of the thermodynamical state of the outer system itself, without the influence of the presence of the droplet, small compared with the dimensions of the vapour volume corresponding to the droplet. In any case the variation of the state of the outer system is of a smaller order compared to the variation of the state of the droplet during the observation time of evolution of the droplet itself.

The equations of evolution of  $R$  and  $T$  can be obtained by equating the mass and energy fluxes respectively to the following quantities:

$$\frac{d}{dt} (\frac{4}{3}\pi R^3 n_i \cdot m) = \Phi_N, \quad (18)$$

$$\frac{4}{3}\pi R^3 n_i \cdot m c_s \frac{dT}{dt} + (2 + \delta) kT \frac{d}{dt} (\frac{4}{3}\pi R^3 n_i) - L(T) \frac{d}{dt} (\frac{4}{3}\pi R^3 n_i \cdot m) = \Phi_E. \quad (19)$$

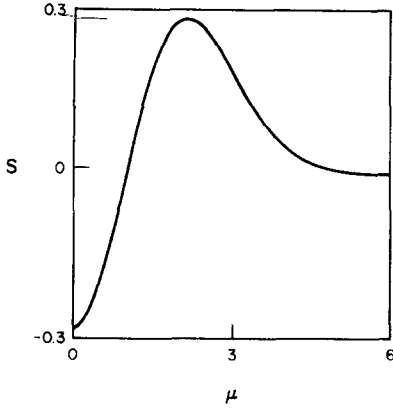


FIG. 1.  $s$  vs  $\mu$ .

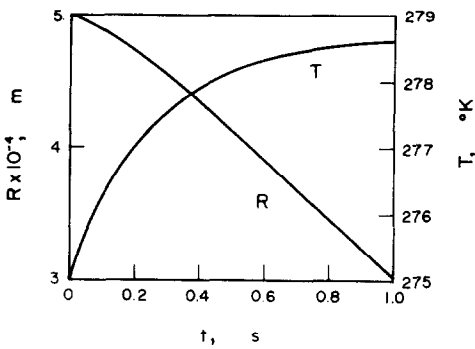


FIG. 2. Evolution of the radius and temperature of a spherical droplet ( $T_a = 290$  K,  $\gamma_0 = 0.8$ ).

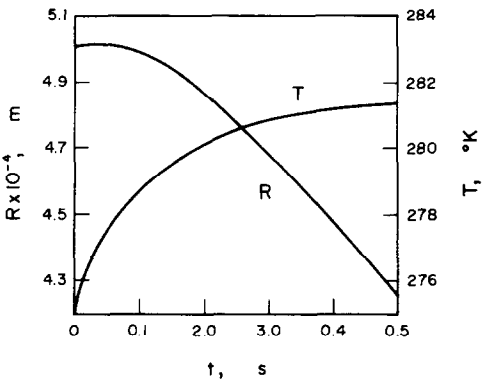


FIG. 3. Evolution of the radius and temperature of a spherical droplet ( $T_a = 290$  K,  $\gamma_0 = 1.2$ ).

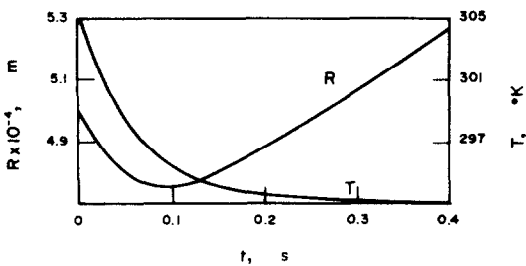


FIG. 4. Evolution of the radius and temperature of a spherical droplet ( $T_a = 290$  K,  $\gamma_0 = 0.8$ ).

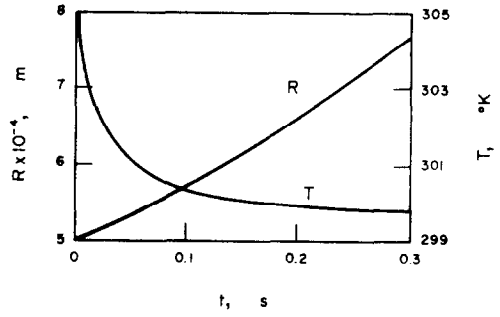


FIG. 5. Evolution of the radius and temperature of a spherical droplet ( $T_a = 290$  K,  $\gamma_0 = 1.2$ ).

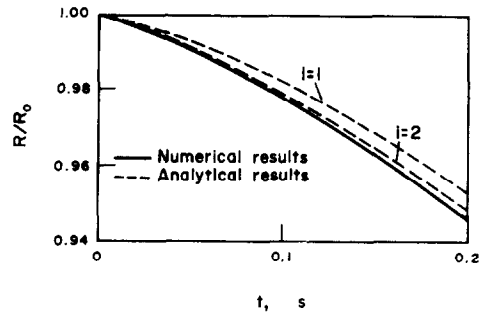


FIG. 6. Comparisons between analytical and numerical results:  $R/R_0$  vs  $t$ .

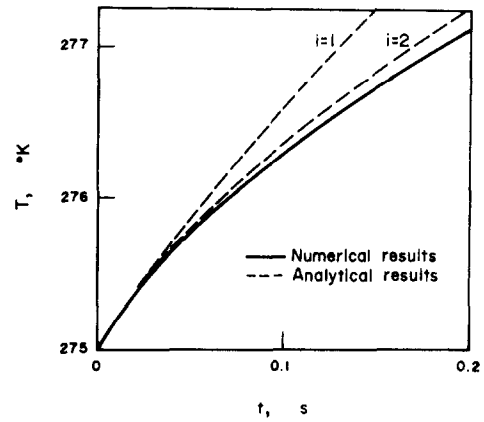


FIG. 7. Comparisons between analytical and numerical results:  $T$  vs  $t$ .

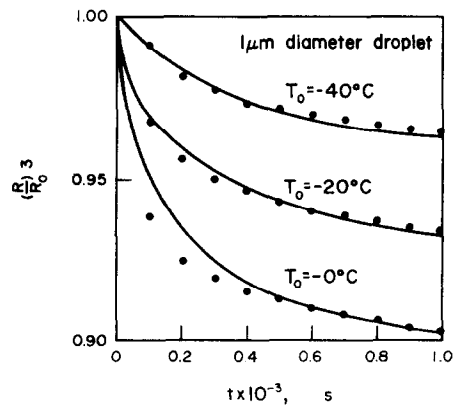


FIG. 8. Comparisons between theoretical and experimental results (ref. [11]).

In particular equation (18) takes into account the time variation of the mass into a spherical volume, while equation (19) takes into account the time variations of heat and collisional energy due to changes of temperature, volume and latent heat of vaporization of a spherical droplet.

The analytical manipulation of equations (16–19) gives the following ordinary differential equations:

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}), \quad \mathbf{x} = (R, T), \quad \mathbf{F} = (F_1, F_2), \quad (20)$$

$$\frac{dR}{dt} = F_1 = F_1(T), \quad (21a)$$

$$\frac{dT}{dt} = F_2 = F_2(R, T) = \frac{g(T)}{R}, \quad (21b)$$

where:

$$F_1 = \frac{1}{2(\pi)^{1/2}} (2k/m \cdot T)^{1/2} v \cdot \phi(\psi_N - 1)$$

$$g = \frac{3}{2(\pi)^{1/2}} v \cdot (2k/m \cdot T)^{1/2} T \left\{ \phi(\psi_N - 1) \cdot \frac{L}{c_s T} + (\psi_E - \psi_N)(2 + \delta) \cdot \frac{k}{mc_s} \right\}$$

$$L = L(T), \quad v = v(T) = n_e/n_t,$$

$$\phi = \phi(T) \text{ given by equation (10);}$$

$$\psi_N = \psi_N(T) \text{ given by equation (13);}$$

$$\psi_E = \psi_E(T) \text{ given by equation (14).}$$

The domain  $D$  of the variables  $(R, T)$  is defined as follows:

$$D_{(R,T)} = (O, R_{\max}) \cdot (T_s, T_c).$$

In  $D$ ,  $F_1$  and  $F_2$  are continuous functions derivable in all variables. The equations (21a) and (21b) are non-linear and must be numerically integrated in order to have the behaviour of  $R$  and  $T$  vs time at given initial conditions.

In the next section, however, some analytical solutions will be proposed and discussed.

#### 4. ANALYTICAL DISCUSSION ON THE MODEL EQUATION $\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x})$ AND COMPARISONS BETWEEN THEORY AND EXPERIMENT

The differential equation (20) which describes the model of the time-evolution of a spherical droplet can be numerically treated with known techniques of integration (see ref. [13]).

In particular let us firstly remark the following property:

Property I: if  $R > 0$  the differential equation (20) is locally Lipschitzian [13] in the sense that:

$$R > 0 \Rightarrow \forall \mathbf{x}_1, \mathbf{x}_2, t: \quad \|\mathbf{F}(\mathbf{x}_1, t) - \mathbf{F}(\mathbf{x}_2, t)\| < a \|\mathbf{x}_1 - \mathbf{x}_2\|.$$

The proof is obvious and consequently the problem of the numerical integration is well-posed.

Taking into account this property, some numerical results are visualized in Figs. (2–5) which show the evolution of  $R$  and  $T$  for four different initial conditions [at fixed  $\tau(0) = \tau_0$  and  $\gamma(0) = \gamma_0$ ] which correspond to different physical situations. In particular:

$$(a) \quad \tau_0 > 1, \quad \gamma_0 < 1 \Rightarrow \forall t: \quad \frac{dR}{dt} < 0, \quad \frac{dT}{dt} > 0 \quad (\text{Fig. 2})$$

this situation corresponds to evaporative heating.

$$(b) \quad \tau_0 > 1, \quad \gamma_0 > 1 \Rightarrow \forall t: \quad \frac{dT}{dt} > 0; \quad \exists t^* > 0: \quad \forall t \in [t^*, \infty] \quad \frac{dR}{dt} < 0 \quad (\text{Fig. 3})$$

this situation corresponds to condensative heating.

$$(c) \quad \tau_0 < 1, \quad \gamma_0 < 1 \Rightarrow \forall t: \quad \frac{dT}{dt} < 0; \quad \exists t^* > 0: \quad \forall t \in [t^*, \infty] \quad \frac{dR}{dt} > 0 \quad (\text{Fig. 4})$$

this situation corresponds to evaporative cooling.

$$(d) \quad \tau_0 < 1, \quad \gamma_0 > 1 \Rightarrow \forall t: \quad \frac{dR}{dt} > 0, \quad \frac{dT}{dt} < 0 \quad (\text{Fig. 5})$$

this situation corresponds to condensative cooling.

The initial conditions play the role defined by the following property:

Property II: the initial condition  $R(0) = R_0$  is a scale operator such that:  $t_1/R_{01} = t_2/R_{02}$  which implies:

$$R(t_1; R_{01}, T_0) \cdot R_{01} = R(t_2; R_{02}, T_0) \cdot R_{02} \quad \text{and} \quad T(t_1; R_{01}, T_0) = T(t_2; R_{02}, T_0).$$

This property is a direct consequence of equation (20) re-written in the form (21a), (21b). Consequently the afore-presented calculations can be extended to a wider range of results.

Besides the above-indicated numerical integration, the problem can be transformed into an equivalent quadrature process, according to the following proposition:

Proposition: the problem  $(R_0, T_0)_{t_0} \rightarrow (R, T)_t$ , defined by the differential equations (20) and (21) is equivalent to a quadrature problem of the type:  $(t_0, R_0)_{T_0} \rightarrow (t, R)_T$ , with  $T$  independent variable and  $(R, t)$  dependent variables.

Proof: let us divide equation (21a) by equation (21b):

$$\frac{dR}{dT} = R \cdot G(T), \quad G(T) = \frac{F_1(T)}{g(T)} \tag{22}$$

the first equation gives:

$$\frac{R}{R_0} = \exp \left\{ \int_{T_0}^T G dT \right\} = \xi(T; T_0) \tag{23a}$$

by substituting this result in equation (21b) and by integrating:

$$(t - t_0) = \int_{T_0}^T H dT = \zeta(T; T_0); \quad H(T) = R_0 \cdot \xi(T; T_0) / g(T) \tag{23b}$$

which proves the proposition.

Remark I: the transformation of problem (20) into a quadrature problem gives a direct information of the bounds of approximation on  $R$  and  $T$ , according to the quadrature algorithm [13].

Remark II: the quadrature indicated by the proposition can lead to analytical solutions of problem (20) in terms of an expansion of  $G$  and  $H$ , and consequently to an analytical integration, if  $G$  and  $H$  are  $n$ -derivable functions of  $T$ :

$$G(T) \cong G(T_0) + \sum_1^n \frac{d^i G}{dT^i} \Big|_{T=T_0} \frac{(T - T_0)^i}{i!} \tag{24a}$$

$$H(T) \cong H(T_0) + \sum_1^n \frac{d^i H}{dT^i} \Big|_{T=T_0} \frac{(T - T_0)^i}{i!}. \tag{24b}$$

According to this expansion equations (23a), (23b) become:

$$\frac{R}{R_0} \cong \exp \left\{ G(T_0) \cdot (T - T_0) + \sum_1^n \frac{d^i G}{dT^i} \Big|_{T=T_0} \frac{(T - T_0)^{i+1}}{(i + 1)!} \right\} \tag{25a}$$

$$(t - t_0) \cong H(T_0) \cdot (T - T_0) + \sum_1^n \frac{d^i H}{dT^i} \Big|_{T=T_0} \frac{(T - T_0)^{i+1}}{(i + 1)!}. \tag{25b}$$

Remark III: if the system is not close to equilibrium conditions ( $\psi_N, \psi_E \neq 1$ ), the derivability of  $G$  and  $H$  is directly verified according to the structure of the auxiliary functions appearing in  $G$  and  $H$ .

In the author's opinion a numerical testing of the procedure can show the convergence of the solution in terms of power expansion; an analytical discussion of the convergence of the serial expansion would introduce a large amount of analytical and numerical work unnecessary towards the real objectives of this paper.

The results of this testing are indicated in Figs. (6 and 7), which show respectively  $R/R_0$  and  $T$  vs  $t$  in the above-mentioned case (a). The dotted lines represent the results of the quadrature process for a truncated expansion of  $G$  and  $H$ , while the black lines represent the correspondent numerical results already shown in Fig. (2). Let us comment that even a second order expansion gives successful results.

Finally in Fig. (8) some comparisons, at different initial temperatures, between the proposed theory and the experimental results of ref. [11], for a water droplet of  $1 \mu\text{m}$  diameter injected into vacuum, are shown. Let us remark that this comparison is quite successful, also considering that the chosen environmental conditions are the limit ones for the proposed model.

## 5. DISCUSSION

In the present work a heat and mass transfer model at the interface of a liquid–vapour system has been proposed starting from a general theory of phase transition [9]. The model has been mathematically determined with the aid of a set of auxiliary functions  $\psi_N$ ,  $\psi_E$  and  $s$  by imposing some limiting conditions due to the physical behaviour of the system [see points (1), (2), (3), (f), (g) and (h) of the second section]. The said model has been then applied to the analysis of the time-evolution of a spherical droplet (in non-equilibrium conditions) in a vapour environment.

Let us remark that the proposed model can be extended to a wider range of physical situations of the system. In particular:

- (1) if the liquid–phase has a motion with velocity  $V$  in the vapour medium, this effect can be taken into account by changing equation (15) in the following form:

$$s = \frac{1}{2(\pi)^{1/2}} (\mu^2 - 1) \cdot \exp\{-\mu^2/3.62\} + V/(2k/m \cdot T_a)^{1/2}; \quad (26)$$

- (2) it may be taken into account a time variation of the state variables of the outer ambient,  $n_a = n_a(t)$  and  $T_a = T_a(t)$ , as already mentioned in section 3.

Let us also remark that it is important to construct evolution models in terms of an ordinary differential equation. This is because in the most of the physical problems the system is not constituted by only one droplet but by a large number of them, and therefore also the distribution function on the state of the object must be determined (see the discussion in [4] referred to the case of droplets in rarefied environment). In particular this model has been applied for the study of the dynamics of fog, see [14]; in fact for the study of such a system it is necessary to know the law of evolution of the droplets belonging to the system itself.

Finally let us note that the problem of time-evolution of liquid droplets is an open area of research, as also indicated in very recent contributions such as [15], and the proposed model can contribute to construct a valid theoretical basis for the related class of physical problems in fluid dynamics.

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## MODELE PHYSICO-MATHEMATIQUE DE L'EVOLUTION TEMPORELLE DE GOUTTELETTES DANS UN ENVIRONNEMENT DE VAPEUR

**Résumé**—Dans cette étude, on propose un modèle physico-mathématique du mécanisme de transfert de masse et d'énergie au travers d'une interface liquide–vapeur.

En utilisant ce modèle et les lois de conservation de la masse et de l'énergie, on a établi les équations d'évolution de l'état d'une goutte dans un milieu continu de vapeur. On aboutit à des équations différentielles non-linéaires qui ont été intégrées numériquement; les résultats obtenus sont comparés à des résultats expérimentaux d'autres auteurs.

ZEITLICHE AUSBINDUNG SPHÄRISCHER TROPFEN IN EINER DAMPF-ATMOSPHERE.  
EIN PHYSIKALISCH-MATHEMATISCHES MODELL

**Zusammenfassung**—In der vorliegenden Arbeit wird ein physikalisch-mathematisches Modell für den Wärme- und Stoffübergangsvorgang an der Grenzfläche eines Dampf-Flüssigkeitssystems vorgeschlagen. Auf der Grundlage dieses Modells und mit Hilfe der Erhaltungssätze von Massen- und Energieströmen werden die Gleichungen für die Ausbildung des Zustandes eines einzelnen Tropfens in einem Kontinuum bestimmt. Diese Gleichungen sind gewöhnliche, nichtlineare Differentialgleichungen, die numerisch integriert wurden, um Vergleiche mit den experimentellen Ergebnissen anderer Autoren zu ermöglichen.

РОСТ СФЕРИЧЕСКИХ КАПЕЛЬ В СРЕДЕ ПАРА. ФИЗИКО-МАТЕМАТИЧЕСКАЯ  
МОДЕЛЬ

**Аннотация**— В статье предложена физико-математическая модель процесса тепло- и массо-переноса на границе раздела системы жидкость-пар. На основе этой модели и с помощью уравнений сохранения потоков массы и энергии выведены эволюционные уравнения состояния для единичной капли в сплошной среде. Эти уравнения, являющиеся обычными нелинейными дифференциальными уравнениями, были проинтегрированы численно для сравнения с экспериментальными данными других авторов.